Math 121
Homework #4
Spring 2007
Due Monday May 7

Page 82-- # 8, 9, 10

Also:

1. A <u>compact exhaustion</u> of a metric space X is a sequence $K_1, K_2, K_3, \ldots$ of compact subsets of X such that

1. $K_j \subset$ interior of K_{j+1} , all j

2. $\cup K_j = X$

Show that if $\{f_i\}$ is a sequence of functions from X to $\mathbb{R}$ such that for each j, the sequence $\{f_i|K_j : i=1,2,3\ldots\}$ is bounded and equicontinuous, then there is a subsequence of $\{f_i\}$ which converges uniformly on every compact subset of X. (Suggestion: Use a "diagonal process" to get uniform convergence on each K_j and note that if $C \subset X$, C compact, then $C \subset$ some $K_{j.)}$

2. Suppose F(x,y,z) is a continuously differentiable function on $\mathbb{R}^3$ with f(0,0,0)=0 and $\left.\frac{\partial f}{\partial z}\right|_{(0,0,0)} \neq 0$. Show that there is an $\varepsilon>0$ such that, for each (x,y) with $x^2+y^2<\varepsilon$, there is a **z** (depending on x and y) with f(x,y,z)=0. Give an example to show that the condition $\left.\frac{\partial f}{\partial z}\right|_{(0,0,0)} \neq 0$ cannot de dropped (in general) and have the conclusion remain true.